# From structural form to state-space form

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RESEARCH MEMORANDUM

From structural form to state-space form

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## Abstract

Starting from an econometric model in structural form, we show how it can be transformed into state-space form. Three possibilities are reviewed and for each we state the implications for:

- the dimension of the state vector, hence the practical use when implemented
- the applicability for optimal control techniques

An existing macro-econometric model serves as an illustration.



## Key words

Macro-econometric models, linear systems, optimal control.

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## 1. Introduction

There is a growing interest for the use of optimal control techniques in policy evaluation with large macro-econometric models. The tradition of econometric modelling leads to models in structural, reduced and final form. The tradition of optimal control theory leads to algorithms which are applicable to models in state-space form. Not so much attention has been paid to the problem of transforming a model in structural form to a model in state-space form. Mostly, the transformation procedure which was presented in Chow [2], p. 153, is used. This transformation procedure is simple and clear. However, it generally leads to models with a very high dimension for the state-space. This is undesirable, because optimal control experiments become costly regarding computer storage space and running time. Moreover, the accuracy of the application results might be harmed by large dimensions.

By the name of realization theory much attention is devoted - in systems and control literature - to the problem of transforming input/output models to state-space models (see e.g. Kailath [3]). The developed algorithms mostly guarantee a minimal realization. A minimal realization is a model in state-space form with the smallest possible dimension, which still describes the input/output behaviour of the original model. It will be clear, that the algorithms can also be applied to state-space models in order to reduce the dimension, while the input/output behaviour is preserved.

The final form of an econometric model describes the input/output behaviour. The input consists of time series for the instruments. It is assumed that the time series for the non-controllable exogenous variables are known beforehand. The output consists of time series for endogenous variables. If one is only interested in the behaviour of certain so-called objective variables (targets), a considerable reduction could be achieved by identifying the output with time series for these objective variables. If one is satisfied with a state-space model which describes the input/output behaviour not precisely but - in some sense - good enough, an even further reduction could be obtained. Especially the idea of a balanced realization is attractive (see Moore [5]).

A minimal realization, which can be found by means of a realization algorithm, serves the purpose of the smallest possible dimension. However, generally state variables result which do not have an economic meaning any longer.

In this paper two procedures are suggested for the transformation of a model in structural form to a model in state-space form. Both procedures are algebraic manipulations. Firstly, the "Chow" transformation procedure is taken as a starting point. The advantage of this procedure is that the state variables remain economic variables. This facilitates the interpretation of application results. Some simple tricks generally lead to a substantial reduction in dimension, while this advantage is preserved. The disadvantage is, that the dimension of the state vector remains unnecessarily high. Moreover, it will be shown that this transformation procedure does not yield a realization, which can be recognized as a linear system, as defined in system theory. This implies, that concepts and techniques from systems and control theory, under which (balanced) realization algorithms, are not automatically applicable.

Secondly, a transformation procedure, analogous to the so-called "observable canonical form" transformation, is taken as a starting point (see Aoki [1], p. 22 ff.). The advantages and disadvantages of the "Aoki" procedure are just opposite to those of the "Chow" procedure. In the last section of the paper the consequences of both procedures for the model Mini-Interplay and for the application of optimal control techniques are discussed. Mini-Interplay is a model for two Common Market countries (see Plasmans [6], Merbis [4] or de Zeeuw [7]).

## 2. Structural, reduced and state-space form

In this section definitions are given of an econometric model in structural, reduced and state-space form. Only deterministic models will be considered here. The structural and reduced form are autoregressive, moving average (ARMA) schemes. The state-space form is a linear system. Under a non-singularity condition, the structural form can be transformed to the reduced form. Throughout this paper it will be assumed that this non-singularity condition holds. The following symbols are used for the different types of variables:

| | |
|---|---|
| $y : T \to R^k$ | endogenous variables |
| $u : T \to R^m$ | instruments |
| $x : T \to R^n$ | state variables |
| $z : T \to R^r$ | non-controllable exogenous variables |

where $T := \{t_0, t_0+1, \ldots\}$ is the time axis.

Definition 2.1.

An econometric model in structural form, denoted by ARMAs(p,q), is given by:

$$\begin{aligned} y(t) = {} & \hat{A}_0\, y(t) + \hat{A}_1\, y(t-1) + \ldots + \hat{A}_p\, y(t-p) \\ & + \hat{B}_0\, u(t) + \hat{B}_1\, u(t-1) + \ldots + \hat{B}_q\, u(t-q) + \hat{F}\, z(t). \end{aligned} \tag{2.1}$$

Definition 2.2.

An econometric model in reduced form, denoted by ARMA(p,q), is given by:

$$\begin{aligned} y(t) = {} & A_1\, y(t-1) + A_2\, y(t-2) + \ldots + A_p\, y(t-p) \\ & + B_0\, u(t) + B_1\, u(t-1) + \ldots + B_q\, u(t-q) + F\, z(t). \end{aligned} \tag{2.2}$$

It will be clear that, if $(I-\hat{A}_0)$ is non-singular, an ARMAs(p,q) model can be transformed to an ARMA(p,q) model, where

$$A_i := (I-\hat{A}_0)^{-1}\, \hat{A}_i, \qquad i = 1,2,\ldots,p,$$

$$B_j := (I-\hat{A}_0)^{-1}\, \hat{B}_j, \qquad j = 0,1,\ldots,q,$$

$$F := (I-\hat{A}_0)^{-1} \hat{F} .$$

Definition 2.3.

An econometric model in state-space form or a linear system, denoted by $\Sigma(A,B,C,D,E)$, is given by:

$$\begin{aligned} x(t+1) &= A\,x(t) + B\,u(t) + E\,z(t) \qquad \text{(state-equation)} \\ y(t) &= C\,x(t) + D\,u(t). \qquad \text{(output-equation)} \end{aligned} \tag{2.3}$$

The ARMA(p,q) model (2.2) induces an input/output behaviour between the inputs u and z and the output y. The realization problem is to find a linear system $\Sigma(A,B,C,D,E)$ (2.3), which induces the same input/output behaviour, with a small state dimension. This problem will be dealt with in sections 3 and 4. It should be noted, that the ARMA(p,q) model generally displays an instantaneous coupling between y(t) and u(t), whereas the linear system $\Sigma(A,B,C,D,E)$ only displays this instantaneous coupling if $D \neq 0$.

## 3. The "Chow" procedure

The transformation of an ARMA(p,q) model to state-space form, which is the most popular one in economic literature, can be found in Chow [2], p. 153. This transformation procedure leads to $D = 0$. This seems to be contradictory with the remark at the end of section 2. However, the Chow procedure does not lead to a linear system in the strict sense. In the state-equation a time shift occurs. In the standard axiomatics of a system the input at time t influences the state at time $\tau$, $\tau > t$, whereas in the realization resulting from the Chow transformation the input at time t influences the state at time $\tau$, $\tau \geqq t$. The latter aspect is not as harmless as it might look at first sight. It implies, that results from system and control theory are not automatically applicable to the Chow transformation.

This section can be split into four parts. Firstly, in theorem 3.1 the Chow transformation is presented. Secondly, the dimension of the state is discussed. Thirdly, an example is given as a warning against the automatical application of system theory. Finally, it is shown by which simple manipulation the Chow procedure could lead to a linear system.

Theorem 3.1.

An ARMA(p,q) model can be transformed to

$$\begin{aligned} x(t) &= A\ x(t-1) + B\ u(t) + E\ z(t) \\ y(t) &= C\ x(t)\ , \end{aligned} \tag{3.1}$$

where

$$x(t) := [y'(t)\ \dots\ y'(t-p+1)\ u'(t)\ \dots\ u'(t-q+1)]' ;$$

$$A := \left[\begin{array}{ccc|ccc} A_1 & \dots & A_p & B_1 & \dots & B_q \\ & & 0 & & & \\ & I & \vdots & & 0 & \\ & & 0 & & & \\ \hline & & & 0 & \dots & 0 \\ & & & & & 0 \\ & 0 & & & I & \vdots \\ & & & & & 0 \end{array}\right] ; \quad B := \left[\begin{array}{c} B_0 \\ 0 \\ \vdots \\ 0 \\ \hline I \\ 0 \\ \vdots \\ 0 \end{array}\right] ;$$

$$C := [I \quad 0 \dots 0] ;$$

$$E := [F \quad 0 \dots 0]' .$$

Proof

By definition of the state vector x(t), the proof is immediately clear.

Q.E.D.

The dimension of the state is $pk + qm$. This figure depends on the dimension of the endogenous vector $k$, the dimension of the instrumental vector $m$ and the numbers of lags $p$ and $q$. There are two immediate ways in which this dimension can be reduced. Firstly, not all variables have to be stacked into the state vector up to the maximum numbers of lags $p$ and $q$: the variables only have to be stacked up to their own maximum lag. Secondly, elimination of endogenous variables, which does not lead to an increase in the maximum lags $p$ and $q$, diminishes the dimension of the endogenous vector $k$ and, hence, diminishes the dimension of the state vector. For example, most static definitional equations may generally be eliminated.

An important result of system theory is that a realization is minimal if it is both reachable and observable. A system is reachable, if each state can be reached from the zero state in a finite number of steps. A system is observable, if the value of the state is uniquely determined by the input and the output. For a linear system both properties can be checked by means of rank conditions on matrices. A linear system is reachable, if the matrix

$$[B \quad AB \ \ldots \ A^{n-1}B]$$

has full rank. A linear system is observable, if the matrix

$$[C' \quad A'C' \ \ldots \ A'^{n-1}C']$$

has full rank (see e.g. Aoki [1] or Kailath [3]). By means of an example it is shown that the Chow transformation can lead to a reachable pair $(A,B)$ and an observable pair $(A,C)$, whereas a realization of the ARMA model can be found with a smaller dimension for the state vector.

Example 3.1.

Consider the ARMA(1,1) model with $k = m = 1$:

$$y(t) = a_1\ y(t-1) + b_0\ u(t) + b_1\ u(t-1).$$

The Chow transformation yields

$$x(t) = A\ x(t-1) + B\ u(t)$$

$$y(t) = C\ x(t)\ ,$$

where

$$x(t) := \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \quad ;$$

$$A := \begin{bmatrix} a_1 & b_1 \\ 0 & 0 \end{bmatrix} \; ; \quad B := \begin{bmatrix} b_0 \\ 1 \end{bmatrix} \; ; \quad C := [1 \quad 0]\ .$$

For almost all values of $a_1$, $b_0$ and $b_1$ $(A,B)$ is reachable and $(A,C)$ is observable, because

$$\begin{bmatrix} b_0 & a_1 b_0 + b_1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 \\ a_1 & b_1 \end{bmatrix}$$

generally have full rank.

However, a realization with state dimension 1 can be constructed:

$$x(t+1) = a_1\ x(t) + (a_1 b_0 + b_1)\ u(t)$$

$$y(t) \quad = \quad x(t) + b_0\ u(t).$$

where $\quad x(t) := y(t) - b_0\ u(t).$ □

Example 2.1 demonstrates in fact how by a simple manipulation the Chow procedure could lead to a linear system. The precise formulation is given in theorem 3.2. This theorem only applies to ARMA(s,s) models. That is to say, the degrees of the autoregressive (AR) part and the moving average (MA) part have to be equal. However, an ARMA(p,q) model can easily be transformed to an ARMA(s,s) model, where $s := \max(p,q)$. If $p = q$, this statement is trivial. If $p > q$, then define

$$B_{q+1} = \dots = B_p = 0$$

and the statement is trivial. If $p < q$, then define

$$A_{p+1} = \dots = A_q = 0$$

and the statement is trivial.

<u>Theorem 3.2</u>.

An ARMA(s,s) model can be transformed to the linear system

$$\begin{aligned} x(t+1) &= A\, x(t) + B\, u(t) + E\, z(t+1) \\ y(t) &= C\, x(t) + D\, u(t) \end{aligned} \qquad (3.2)$$

where

$$x(t) := [\hat{y}'(t) \; \dots \; \hat{y}'(t-s+1) \; u'(t-1) \; \dots \; u'(t-s+1)]';$$

$$\hat{y}(t) := y(t) - B_0\, u(t);$$

$$A := \left[\begin{array}{ccc|ccc} A_1 & \dots & A_s & \hat{B}_2 & \dots & \hat{B}_s \\ & & 0 & & & \\ & I & \vdots & & 0 & \\ & & 0 & & & \\ \hline & & & 0 & \dots & 0 \\ & & & & & 0 \\ & 0 & & & I & \vdots \\ & & & & & 0 \end{array}\right] ; \quad B := \left[\begin{array}{c} \hat{B}_1 \\ 0 \\ \vdots \\ 0 \\ \hline I \\ 0 \\ \vdots \\ 0 \end{array}\right] ;$$

$$\hat{B}_i := B_i + A_i\ B_0, \quad i = 1,2,\ldots,p\ ;$$

$$C \ := [I \ \ 0 \ \ldots \ 0]\ ;$$

$$D \ := B_0\ ;$$

$$E \ := [F \ \ 0 \ \ldots \ 0]'.$$

Proof

The ARMA(s,s) model can be rewritten as:

$$\begin{aligned} y(t+1) - B_0\ u(t+1) = {} & A_1\ (y(t) - B_0\ u(t)) + \ldots \\ & + A_s\ (y(t-s+1) - B_0\ u(t-s+1)) \\ & + (B_1 + A_1\ B_0)\ u(t) + \ldots \\ & + (B_s + A_s\ B_0)\ u(t-s+1) + F\ z(t+1). \end{aligned}$$

The rest of the proof is analogous to the proof of theorem 3.1.

Q.E.D.

Now the dimension of the state vector is $s(k+m) - m$. As opposed to what was found in theorem 3.1 realization (3.2) is a linear system $\Sigma(A,B,C,D,E)$ in the strict sense. As is to be expected in this case, $D \neq 0$. The basic idea of theorem 3.2 is the construction of the state variables $\hat{y}$. A more involved idea for the construction of state variables will be dealt with in the following section.

## 4. The "Aoki" procedure

The basic idea for the realization which will be given in theorem 4.1, can be found in Aoki [1], p. 22 ff. It is known in the literature as the observable canonical form.

At first, an ARMA(p,q) model has to be transformed to an ARMA(s,s) model again, where $s := \max(p,q)$ (see section 3). Furthermore, considerable notational elegance can be achieved with the help of a lag-operator.

Definition 4.1.

A lag-operator $L^p$ is defined by:

$$L^p\ y(t) := y(t-p),\ p = ..,-1,0,1,2,...,\ t \in T.$$

Theorem 4.1.

An ARMA(s,s) model can be transformed to the linear system

$$\begin{aligned} x(t+1) &= A\ x(t) + B\ u(t) + E\ z(t+1) \\ y(t) &= C\ x(t) + D\ u(t) \end{aligned} \tag{4.1}$$

where

$$x(t) := [x_1'(t)\ \dots\ x_s'(t)]'\ ;$$

$$x_1(t) := y(t) - B_0\ u(t)\ ;$$

$$\begin{aligned} x_i(t) := {} & L(A_i\ y(t) + B_i\ u(t) \\ & + L(A_{i+1}\ y(t) + B_{i+1}\ u(t) + \dots \\ & + L(A_s\ y(t) + B_s\ u(t))\dots), \quad i = 2,3,\dots,s\ ; \end{aligned}$$

$$A := \begin{bmatrix} A_1 & & \\ \vdots & I & \\ A_s & 0 \dots & 0 \end{bmatrix} \quad ; \quad B := \begin{bmatrix} \hat{B}_1 \\ \vdots \\ \hat{B}_s \end{bmatrix} \quad ;$$

$$\hat{B}_i := B_i + A_i\ B_0\ , \quad i = 1,2,\dots,s\ ;$$

$$C := [I\ \ 0\ \dots\ 0]\ ;$$

$$D := B_0\ ;$$

$$E := [F\ \ 0\ \dots\ 0]'\ .$$

Proof

The output equation $y(t) = C\ x(t) + D\ u(t)$ will be immediately clear from the definitions of C, D and $x_1(t)$.
The ARMA(s,s) model can be rewritten as

$$\begin{aligned} y(t) - B_0\ u(t) &= L(A_1\ y(t) + B_1\ u(t) \\ &\quad + L(A_2\ y(t) + B_2\ u(t) + \dots \\ &\quad + L(A_s\ y(t) + B_s\ u(t))\dots) + F\ z(t). \end{aligned}$$

It follows, that

$$\begin{aligned} x_1(t+1) &= L^{-1}\ x_1(t) = L^{-1}(y(t) - B_0\ u(t)) = \\ &= A_1\ y(t) + B_1\ u(t) + x_2(t) + F\ z(t+1). \end{aligned}$$

Furthermore,

$$\begin{aligned} x_i(t+1) &= L^{-1}\ x_i(t) = \\ &= A_i\ y(t) + B_i\ u(t) + x_{i+1}(t)\ , \qquad i = 2,3,\dots,s-1, \end{aligned}$$

$$x_s(t+1) = L^{-1}\ x_s(t) = A_s\ y(t) + B_s\ u(t).$$

With $y(t) = x_1(t) - B_0\ u(t)$ and the definitions of A, B and E the state equation will be clear as well.

Q.E.D.

Realization (4.1) is a linear system $\Sigma(A,B,C,D,E)$ with state dimension $n := s.k$. As is to be expected again, $D \neq 0$. The basic idea is the construction of the state variables $x_i$, which can be nicely described with the help of a nested lag-operator format. The evaluation of this result and the results of section 3 will be dealt with in the following section.

## 5. Evaluation

In section 3 and 4, three transformations of an ARMA(p,q) model were presented. The first one ("Chow" (3.1)) does not lead to a linear system $\Sigma(A,B,C,D,E)$, but to a set of first-order difference equations of comparable form with $D = 0$. The second and third one ("Chow" (3.2) and "Aoki" (4.1)) lead to a linear system $\Sigma(A,B,C,D,E)$ with $D \neq 0$.

In systems and control literature realization algorithms have been developed in order to reduce the dimension of the state vector (see e.g. Kailath [3]). If the output variables were only identified with the objective variables, which generally form a subset of the set of endogenous variables, a stronger reduction could be achieved. The same can be said, if one is satisfied with a model which approximately describes the input/output behaviour (see e.g. Moore [5]). These techniques are only applicable to "Chow" (3.2) and "Aoki" (4.1).

In table 5.1 the results of the three transformation procedures as for the dimension of the state vector are summarized. In the same table it is shown what the consequences are for the model Mini-Interplay. Interplay is a model for six Common Market countries (see Plasmans [6]). Mini-Interplay is a sub-model for the Netherlands and the Federal Republic of Germany (see Merbis [4] or de Zeeuw [7]). It is an ARMA(1,1) model with $k = 65$ and $m = 14$. In section 3 it was shortly described how a considerable reduction of dimension can be achieved by "reduced stacking" and "elimination". Table 5.1 presents also the final result, when these tricks are performed with respect to the "Chow" (3.1) transformation of Mini-Interplay. The resulting state-space form is completely described in de Zeeuw [7]. Finally, in table 5.1 the results are given for the "Chow" (3.2) and "Aoki" (4.1) transformations of Mini-Interplay after the same eliminations are performed. These eliminations diminish k from 60 to 23.

Table 5.1: Dimension state-space

| | Abstract | Mini-Interplay |
|---|---|---|
| "Chow" (3.1) | pk + qm | 79 |
| After "reduced stacking" and "elimination" | | 30 |
| "Chow" (3.2) | max(p,q).(k+m)-m | 65 |
| After "elimination" | | 23 |
| "Aoki" (4.1) | max(p,q).k | 65 |
| After "elimination" | | 23 |

The gain of "Aoki" (4.1) with respect to "Chow" (3.2) is (max(p,q)-1).m. This gain does not show in an ARMA(1,1) model. If $p = q$, the gain of "Chow" (3.2) with respect to "Chow" (3.1) is m. It is not yet checked what the dimension of a minimal realization of Mini-Interplay is. This dimension can be found by application of a realization algorithm to either "Chow" (3.2) or "Aoki" (4.1). However, the presented gains with respect to the standard "Chow" (3.1) transformation are considerable already. Moreover, these gains are achieved by relatively simple transformations.

This paper is concluded by a short evaluation of the use of optimal control techniques on the basis of the three transformation procedures.

"Chow" (3.1) has the great advantage, that the state variables are economic variables. This facilitates the interpretation of the results. Moreover, $D = 0$. This implies, that the formulas of the optimal control algorithms become shorter. Especially when a game theory framework is needed - in the case of several policy makers with independent targets -, this second advantage is significant (see de Zeeuw [7]).

"Chow" (3.2) and "Aoki" (4.1) have the great advantage, that the dimension of the state vector and, hence, of the Riccati matrices and tracking vectors is small. Especially after application of a realization algorithm this advantage may be significant.

One final aspect is notable. When the policy makers can make measurements during the planning period, feedback controls have to be considered. Feedback controls are controls which are a function of the output. With the "Chow" (3.1) transformation feedback controls at time t become a function of variables up to time t-1, whereas with the "Chow" (3.2) and "Aoki" (4.1) transformations feedback controls at time t become a function of variables up to time t. Although "Chow" (3.1) is not a system in the strict sense, its "state" is more a memory function than the states of the "Chow" (3.2) and "Aoki" (4.1) realizations. In system theory the concept of state is a memory concept.

The choice for one of the presented transformation procedures should be guided by weighing the advantages and disadvantages and by judging which feedback concept is considered more appropriate.

## 6. Conclusion

Three ways have been considered to transform an econometric model into state-space form. Two consequences of these transformations have been discussed. Firstly, a particular transformation determines the dimension of the resulting state vector, and, secondly, determines the applicability of systems and control techniques, like optimal feedback control.

It appears that both the numbers p and q and the (parsimonious) structure of the ARMA(p,q) model influence the resulting dimension of the state. A considerable reduction can be achieved by elimination of static relationships and "reduced stacking". A macroeconometric model consisting of 65 endogenous variables, serves as an illustration.

An eventual application of a control algorithm might otherwise indicate which transformation is desirable. A choice must be made between a linear system and a state-space format. For the standard, deterministic, linear quadratic control problem the optimal trajectories will be the same in both cases. If the control problem is placed in a stochastic or multi-decisionmakers framework, difficulties might arise. The latter topic needs further research.

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